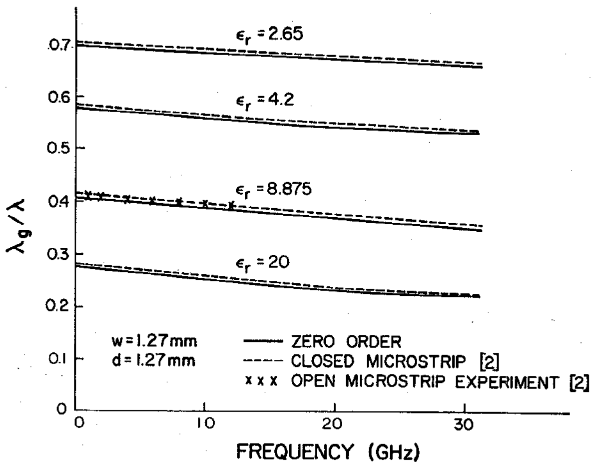

Fig. 4. Normalized guide wavelength versus frequency.

limit agree well with the quasi-TEM solution, except in the case of the first-order approximation.

Finally, it is important to quote typical computation times for this method. The computation time on the CDC G-20 computer (approximately seven to ten times slower than the IBM 360/75) is about 30 s for the zero-order approximation, 120 s for the first-order, and 500 s for the second-order approximation. These times are typical for one point on the curve when the matrix elements given by (11) are accurate to three digits or better.

Conclusions

A numerical method has been presented for obtaining the dispersion relation of the open microstrip lines. The method is based upon application of Galerkin's procedure in the spectral domain. The accuracy of the numerical results obtained by the present method can be improved in a systematic manner by increasing the size of the matrix associated with the characteristic equation. Numerical results reported in this paper have been compared with other available data and experimental results.

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Ridged Waveguide for Planar Microwave Circuits

ROLF O. E. LAGERLÖF

***Abstract*—A TE-mode planar transmission line is analyzed. It has a cross section as a ridged waveguide where the ridges are very thin. It is easily fabricated by photoetching of copper-clad dielectric boards, but can also be made without dielectrics for low-loss applications. Thus, it can be integrated together with other planar transmission lines like, for example, striplines. Besides the simplicity in feeding by stripline, the guide can be made smaller than an ordinary rectangular waveguide. It has applications in filters, resonators, balun-transitions, antenna feeds, etc. The characteristic impedance of the transmission line and its free-space cutoff wavelength are calculated and given in a diagram.**

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The author is with the Division of Network Theory, Chalmers University of Technology, Fack, S-402 20 Gothenburg 5, Sweden.

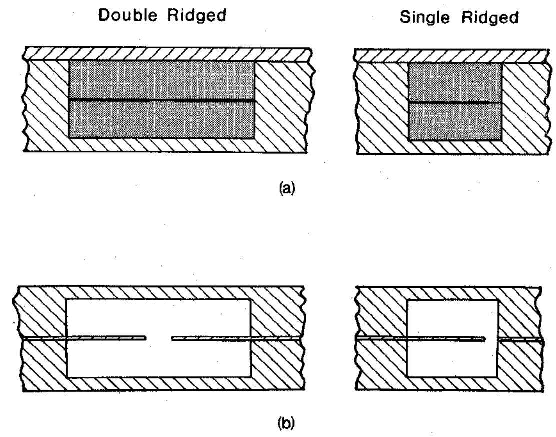

Fig. 1. Practical design of the proposed shielded slotline. (a) Etched copper-clad dielectric boards. (b) Nonsuspended metal ridges in empty guides.

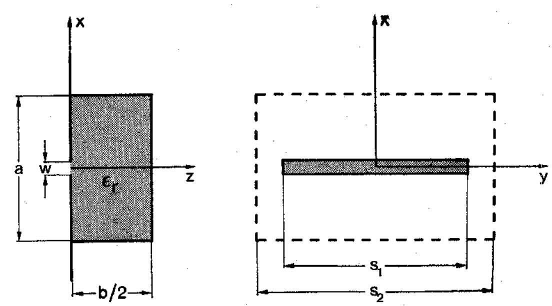

Fig. 2. Cavity-backed slot antenna from which the theory is derived.

In modern microwave circuitry, planar transmission-line technique has assumed an increasing importance. The main reason has been light weight, ease of manufacture, and economy. The two principal planar transmission-line types are the stripline and the microstrip. Recently their "dual forms" have been introduced. Cohn *et al.* [1]–[4] have examined open types of slot lines: a slot in a metal screen with a dielectric substrate on one or both sides of the screen. More or less shielded forms of the slot line have also been investigated [5], [6]. These slot-line types are not only used as transmission lines but also as components in filters, couplers, and ferrite devices [7]–[9]. The completely shielded slot line is closely related to ridged waveguides which are well understood [10]–[12]. However, in the slot-line case, where the ridges are very thin, design information is rare. The purpose of this short paper is to give such information.

The cross section of the shielded slot line is shown in Fig. 1. The ridges or fins form a slot in which the electric field is concentrated. The electric field is uneffected by an electric wall symmetrically placed along the slot and perpendicular to the ridges. Thus there exist two types, the double- and the single-ridged waveguide, with essentially the same characteristics. It is indicated in Fig. 1(a) how shielded slot lines can be fabricated by etching a slot in a copper-clad dielectric board (a stripline board) which together with a nonclad board is placed in a channel milled in a metal block. Precautions shall be taken to ensure good electric contact between the foil ridges and the channel wall, for instance by conductive glue. The ridges can also be made nonsuspended for low-loss application as shown in Fig. 1(b).

The theory for the shielded slot line was achieved as a by-product of an analysis of cavity-backed slot antennas [13]. The cavity part of the antenna admittance at the slot center in Fig. 2 is purely imaginary for a loss-less cavity, and its susceptance is given by

$$B = \frac{32\pi^2 s_1^2}{k_0 s_2 a w^2}\sqrt{\frac{\epsilon_0}{\mu_0}} \sum_{m=0}^{\infty} \frac{k^2 - k_{my}^2}{[\pi^2 - k_{my}^2 s_1^2]^2} \cos^2(k_{my} s_1/2) \cdot \left\{ \frac{w^2}{4}\left(\frac{\coth(k_{m0z} b/2)}{k_{m0z}} + \frac{a}{\pi} \ln \frac{1}{\sin\left(\frac{\pi}{2}\frac{w}{a}\right)} \right) + 2\sum_{n=1}^{\infty} \frac{\sin^2(k_{nx} w/2)}{k_{nx}^2}\left[\frac{\coth(k_{mnz} b/2)}{k_{mnz}} - \frac{1}{k_{nx}}\right]\right\} \quad (1)$$

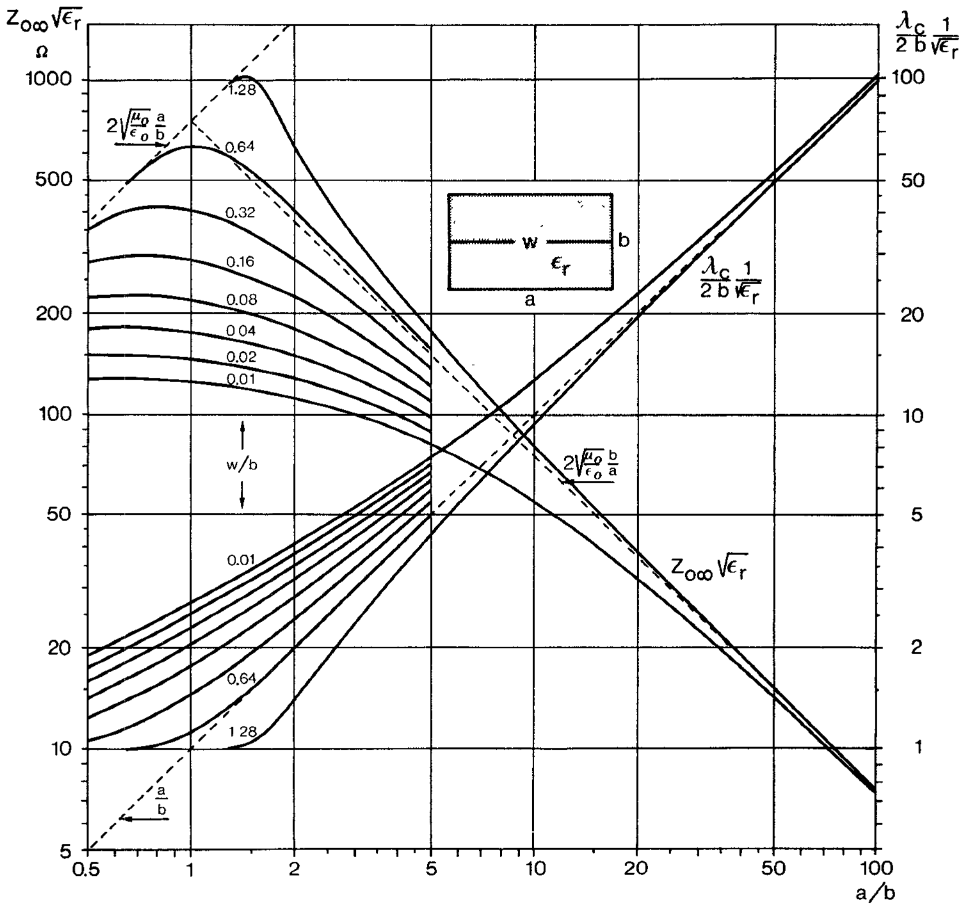

Fig. 3. The characteristic impedance at infinite frequency and the free-space cutoff wavelength for a shielded slot line as functions of the width of the guide for some slot widths. The slot width is doubled between every curve.

where the propagating constants are

$$k_{mnz} = \sqrt{k_{my}^2 + k_{nx}^2 - k^2} \tag{2}$$

$$k_{my} = (2m + 1)\frac{\pi}{s_2} \tag{3}$$

$$k_{nx} = 2n\frac{\pi}{a} \tag{4}$$

$$k = \frac{2\pi}{\lambda_0}\sqrt{\epsilon_r} = k_0\sqrt{\epsilon_r} \tag{5}$$

and the dimensions of the configuration are given by Fig. 2.

Equation (1) has been derived under the assumption that the slot-field distribution along the slot is a half-wave sine function with a method resembling to that used by Cohn [1]. For a slot between two identical cavities, the susceptance is doubled. When the cavities have the same length as the slot, we obtain a half-wave resonator of the waveguide we want to analyze. Its resonance frequency f_0 is obtained from $B = 0$, and the characteristic impedance of the shielded slot line at that frequency is obtained from [1]:

$$Z_0 = \frac{\pi}{\omega}\left.\frac{1}{2\frac{dB}{d\omega}}\frac{v_p}{v_g}\right|_{\omega=\omega_0} \tag{6}$$

where v_p and v_g are the phase and group velocities given by

$$v_p = \frac{\omega_0}{\beta_g} = f_0\lambda_g \tag{7}$$

$$v_g = 1/(d\beta_g/d\omega)\,|_{\omega=\omega_0} = -\lambda_g^2/(d\lambda_g/df)\,|_{f=f_0}. \tag{8}$$

λ_g is the guide wavelength, i.e., double the slot length.

The wave in the guide is a TE mode and thus, we have the following relations [14]:

$$\frac{Z_0}{Z_{0\infty}} = \frac{\lambda_g}{\lambda_0}\sqrt{\epsilon_r} = \frac{1}{\sqrt{1 - (f_c/f_0)^2}} = \frac{1}{\sqrt{1 - (\lambda_0/\lambda_c)^2}}. \tag{9}$$

Since we can calculate Z_0 and f_0 for different λ_g, we can check our formulas with (9). We can also compute the characteristic impedance at infinite frequency, $Z_{0\infty}$, and the cutoff frequency, f_c, or the corresponding free-space cutoff wavelength, λ_c, for the given cross section of the guide. The results of this computation are given in the diagram of Fig. 3. In this, $Z_{0\infty}$ and λ_c can be obtained for an arbitrary cross section of a guide of the double-ridged type and an arbitrary value of the relative dielectric constant, ϵ_r, of the guide filling. When the guide width and the slot width are equal, we get an ordinary rectangular waveguide for which

$$Z_{0\infty} = \sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0}}\,2\frac{a}{b} \tag{10}$$

$$\lambda_c = 2b\sqrt{\epsilon_r}. \tag{11}$$

When the guide width is increased at fixed slot width, the characteristic impedance at infinite frequency and the free-space cutoff wavelength go asymptotically to

$$Z_{0\infty} = \sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0}}\,2\frac{b}{a} \tag{12}$$

$$\lambda_c = 2a\sqrt{\epsilon_r} \tag{13}$$

which is the same as for the fundamental mode in a rectangular waveguide with the width a and the height b. This mode can also propagate in our type of waveguide.

The diagram shows good agreement with Hopfer [11] for $a/b=0.5$ and $a/b=0.9$ which were the only available data for thin ridges.

The analysis assumes infinitely thin ridges. A practical ridge, especially in the (b)-type guide of Fig. 1, has a certain thickness. The effect of that makes $Z_{0\infty}$ smaller and λ_c longer, i.e., the same as for a narrower slot.

The single-ridged waveguide has the same cutoff wavelength but half the characteristic impedance as those of the corresponding double-ridged waveguide.

The feeding of the shielded slot line can be accomplished in several ways. Most interesting is the case when the feeding line is of stripline type. The single-ridged guide is fed by a strip connected straight on the ridge. To feed the double-ridged guide at the ridges, two strips carrying the opposite phase are needed. It can also be fed by a single strip passing perpendicularly under the slot in the same way as slot antennas are fed by striplines [15].

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An Approximate Comparison Between n^+-p-p^+ and p^+-n-n^+ Silicon TRAPATT Diodes

G. I. HADDAD, C. M. LEE, and W. E. SCHROEDER

***Abstract*—The difference in the ionization rates of holes and electrons in Si results in different properties of n^+-p-p^+ and p^+-n-n^+ TRAPATT diodes. An approximate analysis is presented which shows these differences and indicates superior performance in the n^+-p-p^+ structure.**

The avalanche region width in various avalanche-diode structures was considered by Schroeder and Haddad and presented elsewhere [1]. Based on these results, Haddad suggested in 1971 that n^+-p-p^+ Si diodes should be better in the TRAPATT mode than the more commonly employed p^+-n-n^+ ones, especially at lower current densities which are appropriate for CW operation. The reasons for this suggestion were presented elsewhere [2]. Recently, Bierig *et al.* [3] have conducted an experimental comparison between the two types of structures in the TRAPATT mode and have shown that the n^+-p-p^+ diode has superior performance concerning efficiency and power output and were able to obtain CW operation in these structures much more easily than the conventional p^+-n-n^+ ones. The reasons for this, as suggested previously [2] are, as follows.

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The authors are with the Electron Physics Laboratory, Department of Electrical and Computer Engineering, University of Michigan, Ann Arbor, Mich. 48104.

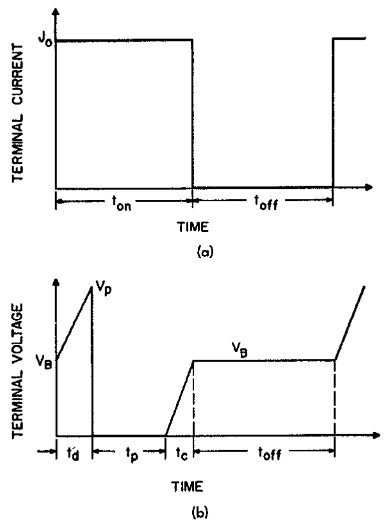

Fig. 1. Device current and voltage waveforms.

1) Since the avalanche region width in the n^+-p-p^+ structure is much narrower than that in the p^+-n-n^+ one [1], especially for high punch-through factor diodes which are appropriate for TRAPATT operation, the IMPATT performance for these diodes should be superior, and thus, it should be easier to generate the overvoltage required for initiation of the trapped plasma mode at current densities which are appropriate for CW operation.

2) As will be shown in the approximate analysis presented here, the delay time and thus the overvoltage are lower for the n^+-p-p^+ structure. This leads to better efficiency, especially at current densities which are appropriate for CW operation.

The analysis presented here should be considered very approximate, and a more exact analysis is presently underway to determine the validity of the various approximations. It is based on three articles which have been published elsewhere. These include the work by Schroeder and Haddad [1] concerning the avalanche region width, the work by Evans [4] and in particular his approximate expression for the delay time and overvoltage, and the work by DeLoach and Scharfetter [5] concerning the recovery time after the initiation of a dense plasma.

In this analysis, the terminal current of the diode is assumed to have the waveform shown in Fig. 1(a), and the corresponding terminal voltage is approximated by the waveform shown in Fig. 1(b), where

t_d the delay time;
t_p the recovery time;
t_c the diode charging time;
t_{on} the on time;
t_{off} the off time.

The diode terminal voltage at the beginning of the cycle is chosen to be the diode dc breakdown voltage V_B. The on time t_{on} is chosen to be $t_{on}=t_d+t_p+t_c$, and the off time t_{off} is chosen to be equal to t_{on}. The waveform shown in Fig. 1 is rather idealized but does give reasonable results at the fundamental frequency and has been employed by others [4]–[6].

To carry out the analysis, the breakdown voltage V_B and the avalanche region width $\bar{w}$ for a particular diode structure are obtained in a manner described elsewhere [1]. Then the delay time t_d and the peak voltage V_p are calculated by solving Evans' [4] equations